On the Construction of the Heavens

Author:
William Herschel, KH, FRS
15 November 1738 – 25 August 1822

Philosophical Transactions of the Royal Society of London
Vol. 75, pp. 213–266, 1785
https://royalsocietypublishing.org/rstl/article-pdf/doi/10.1098/rstl.1785.0012/1461773/rstl.1785.0012.pdf

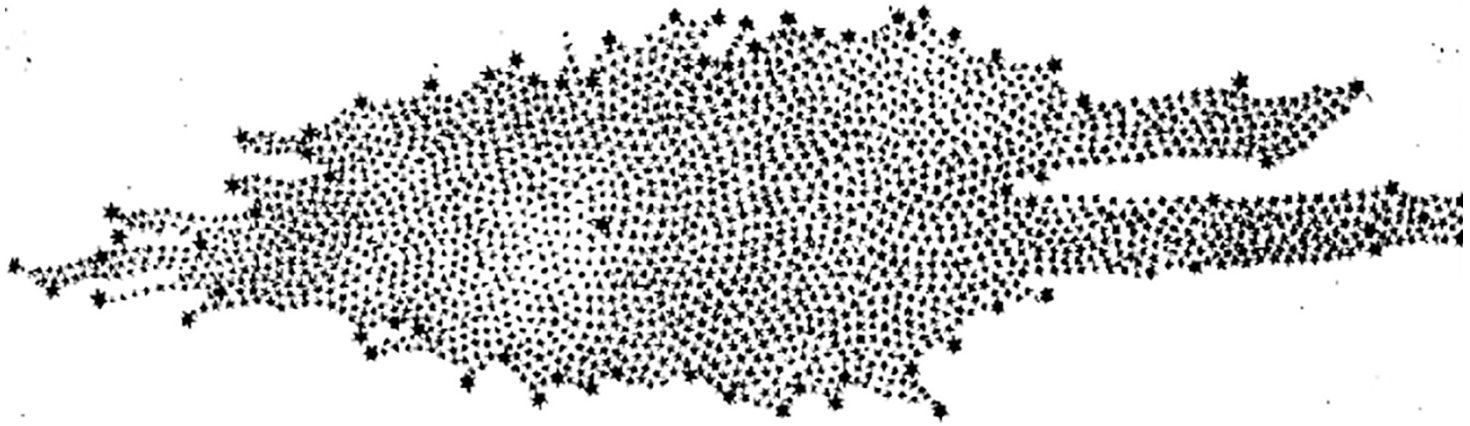

Figure 1: The first drawing of the shape of the Milky Way by William Herschel's in the year 1785

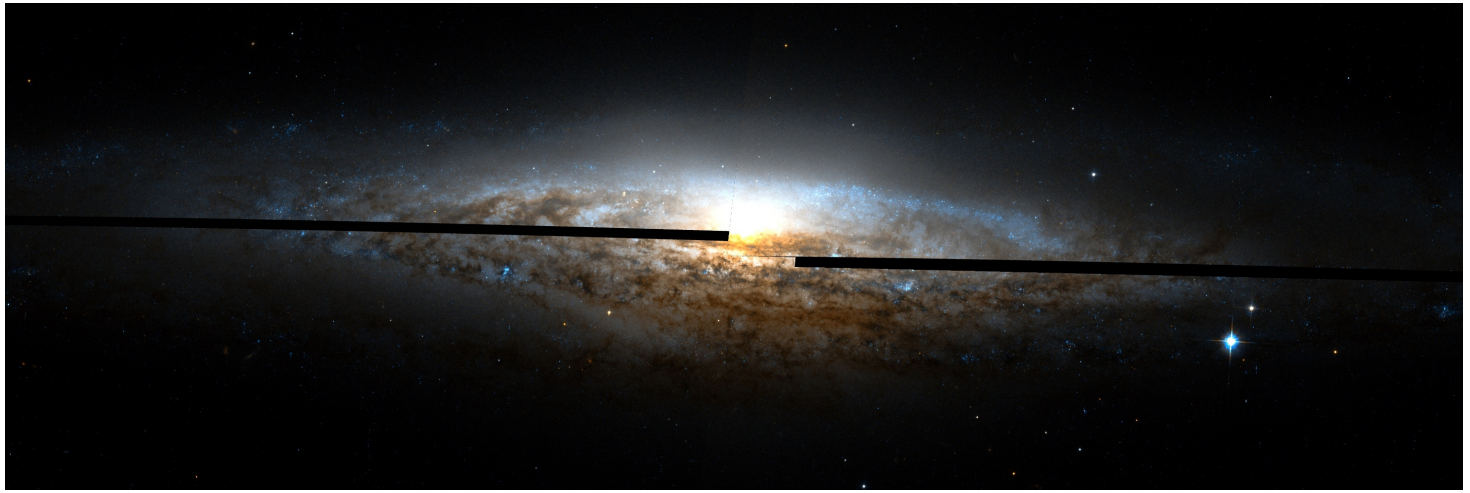

Figure 2: NGC 2683 is an unbarred spiral galaxy discovered by William Herschel on 5 February 1788

Abstract

The subject of the Construction of the Heavens, on which I have so lately ventured to deliver my thoughts to this Society, is of so extensive and important a nature, that we cannot exert too much attention in our endeavours to throw all possible light upon it; I shall, therefore, now attempt to pursue the delineations of which a faint outline was begun in my former paper. By continuing to observe the heavens with my last constructed, and since that time much improved instrument, I am now enabled to bring more confirmation to several parts that were before but weakly supported, and also to offer a few still further extended hints, such as they present themselves to my present view. But first let me mention that, if we would hope to make any progress in an investigation of this delicate nature, we ought to avoid two opposite extremes, of which I can hardly say which is the most dangerous. If we indulge a fanciful imagination and build worlds of our own, we must not wonder at our going wide from the path of truth and nature; but these will vanish like the Cartesian vortices, that soon gave way when better theories were offered. On the other hand, if we add observation to observation, without attempting to draw not only certain conclusions, but also conjectural views from them, we offend against the very end for which only observations ought to be made. I will endeavour to keep a proper medium; but if I should deviate from that, I could with not to fall into the latter error.

1 Introduction

That the milky way is a most extensive stratum of stars of various sizes admits no longer of the lead doubt; and that our sun is actually one of the heavenly bodies belonging to it is as evident. I have now viewed and gaged this shining zone in almost every direction, and find it composed of stars whose number, by the account of these gages, constantly increases and decreases in proportion to its apparent brightness to the naked eye. But in order to develop the ideas of the universe that have been suggested by my late observations, it will be best to take the subject from a point of view at a considerable distance both of space and of time.

2 Theoretical View

Let us then suppose numberless stars of various sizes, scattered over an indefinite portion of space in such a manner as to be almost equally distributed throughout the whole. The laws of attraction, which no doubt extend to the remotest regions of the fixed stars, will operate in such a manner as most probably to produce the following remarkable effects.

3 Formation of Nebulæ

Form I. In the first place, since we have supposed the stars to be of various sizes, it will frequently happen that a star, being considerably larger than its neighbouring ones, will attract them more than they will be attracted by others that are immediately around them; by which means they will be, in time, as it were, condensed about a centre; or, in other words, form themselves into a cluster of stars of almost a globular figure, more or less regularly so, according to the size and original distance of the surrounding stars. The perturbations of these mutual attractions must undoubtedly be very intricate, as we may easily comprehend by considering what Sir Isaac Newton says in the first book of his Principia, in the 38th and following problems; but in order to apply this great authors reasoning of bodies moving in ellipses to such as are here, for a while, supposed to have no other motion than what their mutual gravity has imparted to them, we must suppose the conjugate axes of these ellipses indefinitely diminished, whereby the ellipses will become straight lines.
Form II. The next case, which will also happen almost as frequently as

the former, is where a few stars, though not superior in size to the rest, may chance to be rather nearer each other than the surrounding ones; for here also will be formed a prevailing attraction in the combined centre of gravity of them all, which will occasion the neighbouring stars to draw together; not indeed so as to form a regular or globular figure, but however in such a manner as to be condensed towards the common centre of gravity of the whole irregular cluster. And this construction admits of the utmost variety of shapes, according to the number and situation of the stars which first gave rise to the condensation of the rest.

Form III. From the composition and repeated conjunction of both the foregoing forms, a third may be derived, when many large stars, or combined small ones, are situated in long extended, regular, or crooked rows, hooks, or branches; for they will also draw the surrounding ones, so as to produce figures of condensed stars coarsely similar to the former which gave rise to these condensations.

Form IV. We may likewise admit of still more extensive combinations; when, at the same time that a cluster of stars is forming in one part of space, there may be another collecting in a different, but perhaps not far distant quarter, which may occasion a mutual approach towards their common centre of gravity.

Form V. In the last place, as a natural consequence of the former cases, there will be formed great cavities or vacancies by the retreat of the stars towards the various centres which attracted them; so that upon the whole there is evidently a field of the greatest variety for the mutual and combined attractions of the heavenly bodies to exert themselves in. I shall, therefore, without extending myself farther upon this subject, proceed to a few considerations, that will naturally occur to everyone who may view this subject in the light I have here done.

4 Objections Considered

At first sight then it will seem as if a system, such as it has been displayed in the foregoing paragraphs, would evidently tend to a general destruction, by the shock of one star's falling upon another. It would here be a sufficient answer to say, that if observation should prove this really to be the system of the universe, there is no doubt but that the great Author of it has amply provided for the preservation of the whole, though it should not appear to us in what manner this is effected. But I shall moreover point out several circumstances that do manifestly tend to a general preservation; as, in the first place,

the indefinite extent of the sidereal heavens which must produce a balance that will effectually secure all the great parts of the whole from approaching to each other. There remains then only to see how the particular stars belonging to separate clusters will be preserved from rushing on to their centres of attraction. And here I must observe, that though I have before, by way of rendering the case simpler, considered the stars as being originally at rest, I intended not to exclude projectile forces; and the admission of them will prove such a barrier against the seeming destructive power of attraction as to secure from it all the stars belonging to a cluster, if not for ever, at least for millions of ages. Besides, we ought perhaps to look upon such clusters, and the destruction of now and then a star, in some thousands of ages, as perhaps the very means by which the whole is preserved and renewed. These clusters may be the *Laboratories* of the universe, if I may so express myself, wherein the most salutary remedies for the decay of the whole are prepared.

5 Optical Appearances

From this theoretical view of the heavens, which has been taken, as we observed, from a point not less distant in time than in space, we will now retreat to our own retired station, in one of the planets attending a star in its great combination with numberless others; and in order to investigate what will be the appearances from this contracted situation, let us begin with the naked eye. The stars of the first magnitude being in all probability the nearest, will furnish us with a step to begin our scale; setting off, therefore, with the distance of Sirius or Arkturus, for instance, as unity, we will at present suppose, that those of the second magnitude are at double, and those of the third at treble the distance, and so forth. It is not necessary critically to examine what quantity of light or magnitude of a star entitles it to be estimated of such or such a proportional distance, as the common coarse estimation will answer our present purpose as well; taking it then for granted, that a star of the seventh magnitude is about seven times as far as one of the first, it follows, that an observer, who is enclosed in a globular cluster of stars, and not far from the centre, will never be able, with the naked eye, to see to the end of it: for, since, according to the above estimations, he can only extend his view to about seven times the distance of Sirius, it cannot be expected that his eyes should reach the borders of a cluster which has perhaps not less than fifty stars in depth everywhere around him. The whole universe, therefore, to him will be comprised in a

set of constellations, richly ornamented with scattered stars of all sizes. Or if the united brightness of a neighbouring cluster of stars should, in a remarkable clear night, reach his sight, it will put on the appearance of a finally faint, whitish, nebulous cloud, not to be perceived without the greatest attention. To pass by other situations, let him be placed in a much extended stratum, or branching cluster of millions of stars, such as may fall under the III^{d} form of nebulæ considered in a foregoing paragraph. Here also the heavens will not only be richly scattered over with brilliant constellations, but a shining zone or milky way will be perceived to surround the whole sphere of the heavens, owing to the combined light of those stars which are too small, that is, too remote to be seen. Our observer's sight will be so confined, that he will imagine this single collection of stars, of which he does not even perceive the thousandth part, to be the whole contents of the heavens. Allowing him now the use of a common telescope, he begins to suspect that all the milkiness of the bright path which surrounds the sphere may be owing to stars. He perceives a few clusters of them in various parts of the heavens, and finds also that there are a kind of nebulous patches; but still his views are not extended so far as to reach to the end of the stratum in which he is situated, so that he looks upon these patches as belonging to that system which to him seems to comprehend every celestial object. He now increases his power of vision, and, applying himself to a close observation, finds that the milky way is indeed no other than a collection of very small stars. He perceives that those objects which had been called nebulæ are evidently nothing but clusters of stars. He finds their number increase upon him, and when he resolves one nebula into stars he discovers ten new ones which he cannot resolve. He then forms the idea of immense strata of fixed stars, of clusters of stars and of nebulæ (a); till, going on with such interesting observations, he now perceives that all these appearances must naturally arise from the confined situation in which we are placed. *Confined* it may justly be called, though in no less a space than what before appeared to be the whole region of the fixed stars; but which now has assumed the shape of a crookedly branching nebula; not, indeed, one of the least, but perhaps very far from being the most considerable of those numberless clusters that enter into the construction of the heavens.

6 Result of Observations

I shall now endeavour to shew, that the theoretical view of the system of the universe, which has been exposed in the foregoing part of this paper, is perfectly consistent with facts, and seems to be confirmed and established by a series of observations. It will appear, that many hundreds of nebulæ of the first and second forms are actually to be seen in the heavens, and their places will hereafter be pointed out. Many of the third form will be described, and instances of the fourth related. A few of the cavities mentioned in the fifth will be particularized, though many more have already been observed; so that, upon the whole, I believe, it will be found that the foregoing theoretical view, with all its consequential appearances, as seen by an eye enclosed in one of the nebulæ, is no other than a drawing from nature, wherein the features of the original have been closely copied; and I hope the resemblance will not be called a bad one, when it shall be considered how very limited must be the pencil of an inhabitant of so small and retired a portion of an indefinite system in attempting the picture of so unbounded an extent.

But to proceed to particulars: I shall begin by giving the following Table 1 of gages that have been taken. In the first column is the right ascension (R.A.), and in the second the north polar distance (P.D.), both reduced to the time of FLAMSTEED's Catalogue[1]. In the third are the contents of the heavens, being the result of the gages. The fourth shews from how many fields of view the gages were deduced, which have been ten or more where the number of the stars was not very considerable; but, as it would have taken too much time, in high numbers, to count so many fields, the gages are generally single. Where the stars happened to be uncommonly crowded, no more than half a field was counted, and even sometimes only a quadrant; but then it was always done with the precaution of fixing on some row of stars that would point out the division of the field, so as to prevent any considerable mistake. When five, ten, or more fields are gaged, the polar distance in the second column of the table is that of the middle of the sweep, which was generally from 2 to 2½ degrees in breadth; and, in gaging, a regular distribution of the fields, from the bottom of the sweep to the top, was always strictly attended to. The fifth column contains occasional remarks relating to the gages.

[1]For further Information on the "Atlas Coelestis" by J. FLAMSTEED see `https://dn790009.ca.archive.org/0/items/atlascoelestis00flam/atlascoelestis00flam.pdf` and `https://en.wikipedia.org/wiki/Atlas_Coelestis`.

Table 1: Table of Star-Gages

The gages marked with an asterisk (*) are those by which Figure 3 (*Fig. 4*) (tab. VIII.) has been delineated.

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
0	1	41	78	47	9.9	10	
0	4	55	65	36	20.0	10	
0	7	54	74	13	11.3	10	Most of the stars extremely small.
0	8	24	49	7	60	1	
0	9	52	113	17	4.1	10	*
0	12	52	113	17	3.2	10	*
0	16	48	67	44	11.9	10	
0	21	52	113	17	3.9	10	*
0	22	21	87	10	5.9	10	
0	28	26	46	54	60	1	
0	31	38	46	54	40	1	
0	33	33	65	32	20.4	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
0	34	22	56	38	20	1	
0	35	22	55	38	24	1	
0	36	39	70	32	11.3	10	
0	39	56	78	43	8.1	10	
0	40	29	48	43	60	0.5	
0	44	21	87	10	7.6	10	
0	46	22	69	51	11	10	
0	46	33	65	32	13	10	
0	48	42	58	47	40	1	
0	48	50	58	13	17	1	
0	53	18	67	41	9.8	10	A little hazy.
0	53	40	45	37	73	1	
0	54	10	75	16	13	1	
0	55	10	73	16	14	1	
0	56	4	74	0	15	1	
0	57	52	113	17	3.8	10	*

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
0	59	10	74	25	14	1	
1	0	16	74	16	11.1	10	
1	1	10	74	5	11.2	10	
1	1	18	111	0	5.2	10	Very clear for this altitude.
1	2	52	52	0	28.1	10	Most of the stars very small.
1	3	52	113	17	2.8	10	*
1	4	15	94	52	7.5	10	
1	4	33	65	32	11.0	10	
1	5	55	78	31	9.2	10	
1	7	27	45	23	58	1	
1	12	0	58	37	20	1	
1	12	48	60	19	13	1	
1	13	4	94	50	6.3	10	
1	15	51	48	40	30	1	
1	18	21	48	40	58	1	
1	23	21	48	40	44	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
1	27	30	65	42	12.9	1	
1	31	21	87	7	5.8	10	
1	32	4	94	50	7.3	10	
1	33	10	100	8	6.4	10	
1	33	32	92	35	7.1	10	
1	34	52	60	8	17	1	
1	43	30	65	42	14.4	10	
1	45	24	69	43	7.1	10	
1	48	4	100	12	4.9	10	
1	54	24	76	28	12.1	10	
1	58	55	61	55	15.0	10	
2	4	28	87	5	6.4	10	
2	4	36	78	38	9.3	10	
2	7	12	94	56	7.8	10	
2	8	0	83	3	7.3	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
2	10	4	100	12	4.3	10	
2	11	30	65	45	14.8	10	
2	16	27	110	54	4.2	10	∗
2	19	27	76	24	9.9	10	
2	22	17	45	31	82	1	
2	23	6	60	16	14	1	
2	23	19	113	8	4.2	10	∗
2	24	6	58	30	15	1	
2	27	40	115	21	3.0	10	∗ The situation too low for great accuracy.
2	30	0	94	56	6	10	
2	31	23	76	22	13.8	10	
2	35	14	87	2	5.6	10	
2	38	0	94	56	6.6	10	
2	42	7	61	50	14.8	10	
2	47	32	74	3	11.1	10	Most of the stars exceedingly small.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
2	49	22	92	55	9.0	10	
2	49	30	110	55	6.1	10	*
2	50	0	94	56	6.8	10	
2	54	53	76	22	9.2	10	
2	59	56	81	10	6.1	10	
3	1	53	78	37	4.1	10	
3	1	56	81	10	5.1	10	
3	4	53	78	37	3.5	10	
3	10	20	100	2	6.8	10	
3	11	6	59	29	7.0	5	In a part of the heavens which looks pretty full of stars to the naked eye.
3	13	6	59	29	6.1	10	
3	15	6	59	29	9.4	10	
3	22	57	83	1	10.3	10	
3	23	21	92	49	10.1	10	
3	29	41	46	35	55	1	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
3	35	0	62	1	15	1	About 15 stars generally in the field.
3	35	12	100	3	7.4	10	
3	36	1	113	3	4.9	10	*
3	42	49	46	10	54	1	
3	48	16	99	59	8.1	10	
3	55	11	74	2	11.0	10	
4	1	24	92	48	13.8	10	
4	6	18	82	57	13.4	10	
4	8	31	114	55	4.2	10	*
4	12	41	69	33	15.3	10	And many more, extremely small, suspected.
4	16	34	112	45	6.2	10	*
4	26	34	112	45	8.8	10	*
4	27	11	70	41	25	1	
4	28	41	70	1	17	1	
4	29	5	69	24	30	1	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
4	30	14	99	50	9.7	10	
4	31	19	67	33	15.6	10	
4	32	29	69	2	36	1	
4	33	31	114	55	8.1	10	*
4	42	14	86	27	19.9	10	
4	53	22	72	59	56	1	
4	57	45	83	22	38	1	
4	58	45	84	36	35	1	
5	1	16	69	23	34	1	
5	3	45	83	29	17.7	6	
5	10	52	69	22	74	1	
5	11	22	96	37	24	1	
5	17	22	96	15	8.9	8	
5	18	0	80	46	30	1	About 30 stars in the field, not very exactly gaged.
5	21	7	92	52	19.1	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
5	24	12	66	5	36	1	
5	27	3	68	52	58	1	
5	27	48	110	40	17.7	10	⁎
5	33	4	76	10	65	1	
5	33	12	66	26	86	1	
5	33	17	114	59	13.5	10	⁎
5	34	45	70	33	50	1	
5	36	30	62	1	20 –30	–	From 20 to 30 stars in the fields, not very exactly gaged.
5	37	4	74	26	140	0.5	
5	38	45	70	8	73	1	
5	41	12	66	43	60	1	
5	44	0	116	43	11.5	10	⁎
5	45	30	83	30	50	1	
5	47	34	112	34	19.3	10	⁎

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
5	48	30	62	1	30	1	About 30 stars in the field; not very exactly gaged.
5	48	44	92	51	22.4	5	
5	49	0	80	5	50	1	
5	52	14	93	14	44	1	
5	52	30	83	30	60	1	
5	53	0	80	5	110	1	
5	55	4	92	56	57	1	
5	56	40	70	27	73	1	
5	57	0	80	5	60	1	
5	57	37	110	33	19.6	10	*
5	58	51	88	36	90	1	
5	59	30	83	30	80	1	
6	0	23	86	38	24.1	10	
6	1	0	80	5	70	1	
6	4	0	80	5	90	1	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars		Fields	Memorandums
H.	M.	S.	D.	M.				
6	5	4	67	17	120		0.25	Very unequally scattered.
6	6	14	96	16	52		1	
6	6	30	83	30	80		1	
6	6	30	80	5	70		1	
6	6	38	91	45	54		1	Like the rest, or many such fields.
6	6	40	68	24	56		1	
6	9	0	80	5	74		1	
6	9	34	113	35	26		1	*
6	11	0	62	1	30	–40	1	Between.
6	11	0	80	5	63		1	The least number of stars in the field I could find in this neighbourhood.
6	11	34	112	5	33		1	*
6	11	37	90	15	–		1	About 60 or 70 generally.
6	14	4	68	11	178		0.25	
6	14	38	90	15	77		1	
6	17	45	62	1	50		1	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
6	18	14	96	12	38	1	Very unequally scattered.
6	19	14	93	59	72	1	
6	26	17	114	59	15.9	10	
6	27	14	94	36	132	0.5	*
6	27	32	70	23	50	1	
6	31	48	115	40	40	1	
6	34	44	92	25	94	1	
6	34	55	79	5	50	1	Generally about 50 stars.
6	36	0	94	56	62	1	Twilight.
6	37	15	75	5	70	1	Generally about 70 stars.
6	39	8	99	7	50	1	*
6	40	0	116	43	31.3	10	
6	43	25	79	5	67	1	
6	44	28	100	0	67	1	*
6	49	5	87	21	120	0.5	*
6	49	30	77	31	50	1	Many fields like this.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
6	49	44	92	33	130	0.5	
6	51	8	98	33	78	1	*
6	52	0	116	21	48	1	
6	52	25	79	5	60	1	About 60 stars.
6	52	44	92	59	98	1	
6	54	9	111	11	45	1	*
6	57	8	100	1	34	1	*
6	57	38	98	50	83	1	*
6	58	39	112	48	81	1	*
7	0	25	79	5	70	1	
7	4	0	92	3	102	1	*
7	4	38	98	59	70	1	*
7	5	9	111	11	70	1	*
7	8	9	112	15	62	1	*
7	12	8	100	5	118	0.5	*

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
7	15	38	98	12	112	0.5	⁎
7	19	0	91	51	58	1	⁎
7	20	0	78	59	48	1	
7	25	9	111	21	168	0.25	⁎ One of the richest fields.
7	28	9	112	34	204	0.25	⁎ A field like the rest.
7	33	3	115	28	86	1	
7	41	9	113	26	108	0.5	⁎
7	53	4	86	39	28.3	10	⁎
8	1	4	111	15	80	1	⁎
8	3	4	113	31	66	1	
8	6	38	100	5	40	1	
8	7	38	99	3	45	1	⁎
8	11	8	99	25	24.2	10	⁎
8	12	34	112	15	52	1	⁎
8	22	4	111	30	35	1	⁎

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
8	31	4	112	1	33	1	
8	32	24	112	7	30	1	
8	35	4	112	17	24	1	
8	35	14	111	19	20	1	
8	40	4	111	11	22	1	*
8	45	4	113	22	13	1	
8	46	39	91	26	20.3	10	*
8	48	4	112	23	16.2	10	
8	57	25	66	20	8.3	10	*
9	5	38	91	22	13.8	10	*
9	10	4	115	17	14.0	10	
9	20	4	112	23	15.8	10	
9	20	40	99	12	11.1	10	
9	20	58	88	7	11.5	10	*
9	35	4	112	23	13.0	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
9	38	4	115	17	10.1	10	
9	38	8	90	23	7.9	10	*
9	42	16	86	16	7.7	10	*
9	45	49	112	21	13.2	10	Strong twilight.
10	0	4	115	17	9.1	10	
10	16	8	88	8	7.2	10	*
10	19	32	91	14	6.5	10	
10	25	8	88	8	4.9	10	*
10	26	0	81	41	5.6	7	*
11	4	4	81	38	5.3	6	*
11	7	36	91	14	5.6	10	
11	10	6	115	23	6.5	10	Twilight.
11	16	52	81	38	3.1	8	*
11	20	37	91	17	4.9	10	
11	53	43	81	39	6.0	5	*

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
12	5	6	78	57	2.2	13	⁎
12	30	40	79	3	3.4	11	⁎
12	46	51	81	40	4.6	13	⁎
12	48	19	79	4	3.9	13	⁎
12	53	45	101	45	9.3	10	Twilight.
12	57	8	99	56	8.1	10	Pretty strong day-light.
13	1	19	79	4	3.8	12	⁎
13	17	27	101	45	8.6	10	Twilight.
13	22	49	100	1	8.4	10	Some day-light.
13	27	57	101	45	11.3	10	
13	31	10	75	55	5 –6	1	⁎ Generally about 5 or 6 stars in the field.
13	38	53	104	27	8.5	10	
13	48	49	100	1	9.2	10	Strong twilight.
13	51	27	101	45	10.0	10	
13	55	44	58	11	7.4	10	⁎ Twilight.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
13	57	53	104	27	12.3	10	Most very small.
14	9	49	100	1	11.2	10	Twilight.
14	13	52	113	4	9.7	10	
14	14	57	101	45	8.8	10	
14	24	49	81	53	2.7	6	
14	29	45	100	5	13.3	10	
14	30	7	66	3	8.8	10	∗ All sizes.
14	30	8	80	38	3.5	13	
14	33	22	58	7	8.9	10	∗ Chiefly small.
14	33	52	113	4	10.3	10	
14	39	57	101	45	14.0	10	All sizes.
14	40	36	64	47	6.4	10	
14	44	11	114	54	10.3	10	
14	49	52	113	4	12.8	10	
14	51	14	58	10	9.2	10	∗ Twilight.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
14	52	58	60	41	4.4	10	⁎ Strong Aurora borealis.
14	53	7	66	15	9.0	10	Chiefly large.
14	55	36	64	47	6.6	10	Most very small.
14	59	11	114	54	8.8	10	
15	2	42	62	48	8.3	10	
15	3	7	66	15	9.5	10	
15	4	36	64	47	5.0	10	
15	8	37	113	0	14.1	10	
15	8	45	93	5	9.4	12	Very small.
15	13	42	62	48	8.9	10	
15	15	44	58	17	10.0	10	⁎ Twilight.
15	19	48	60	40	4.9	10	⁎ Strong Aurora borealis, so as to affect the gages.
15	20	0	75	52	9.5	4	
15	21	0	93	5	10.9	12	
15	26	7	81	53	11.0	5	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
15	28	48	99	51	13.1	10	
15	29	7	66	15	10.6	10	All sizes.
15	29	44	58	17	8.9	10	∗ Twilight.
15	32	0	75	51	6	6	
15	33	52	111	32	12.8	10	
15	35	0	75	51	6.5	6	
15	42	2	58	14	13.1	10	∗ Twilight.
15	42	3	116	56	18.6	10	
15	42	53	113	47	32.5	2	The stars too small for the gage.
15	46	30	93	5	10.8	12	
15	48	37	113	0	17.1	10	
15	48	46	63	4	12.4	10	
15	49	52	111	32	18.1	10	The situation so low that it requires attention to see the stars.
15	50	20	114	55	9.2	10	
15	57	3	116	56	7.2	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
16	0	2	58	14	12.2	10	∗ Twilight.
16	0	3	116	56	6.1	10	
16	0	12	114	57	1.6	10	
16	3	12	114	57	2.0	10	
16	4	0	75	43	13	6	All sizes.
16	4	19	113	6	0.5	10	Perfectly clear.
16	4	46	63	4	12.0	10	Most small.
16	4	52	99	57	14.6	10	Moon and twilight.
16	6	28	113	4	0.7	10	Perfectly clear.
16	7	12	66	15	13.3	10	
16	8	6	115	1	3.8	6	
16	8	11	93	9	12.2	12	
16	8	16	116	48	11.6	10	
16	9	28	113	4	1.1	10	Perfectly clear.
16	11	28	113	4	1.4	10	The same.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
16	13	28	113	4	1.8	10	g Serpentarii and 19 Scorpii visible to the naked eye.
16	13	52	58	24	14.2	10	∗ Most small.
16	14	42	63	7	15.1	10	Most very small.
16	15	37	80	40	9.7	12	All sizes.
16	17	28	113	4	4.7	10	
16	20	51	81	57	13.8	6	
16	23	0	73	43	24	1	
16	23	28	113	4	13.6	10	
16	24	11	93	9	13.6	12	Requires attention to be seen.
16	25	7	80	40	14.6	13	
16	27	32	68	23	21.6	10	Twilight.
16	29	16	116	48	50.4	10	
16	30	37	80	40	34	1	
16	31	12	66	15	18.4	10	Strong twilight.
16	32	28	113	4	20.3	10	Most extremely small.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
16	32	52	58	24	15.6	10	⁎ Most small.
16	35	42	63	7	16.5	10	⁎
16	35	48	93	15	18.6	12	All sizes.
16	38	12	66	15	20.1	10	Strong twilight.
16	38	45	107	57	19.9	10	Strong twilight.
16	40	51	113	14	41.1	8	
16	45	32	68	23	19.0	4	Hazy.
16	51	45	107	57	29.8	10	
16	52	22	66	26	16.6	10	Day-light pretty strong.
16	55	42	63	7	26.6	10	⁎ Strong twilight.
17	1	34	58	11	18.8	10	⁎ Strong day-light.
17	3	22	66	26	35	1	⁎ Day-light too strong for gaging.
17	6	36	98	38	13.7	10	Most small, and more suspected.
17	9	30	116	55	7.6	10	
17	9	32	68	23	32.3	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
17	11	10	66	26	38	1	∗ Day-light pretty strong.
17	13	24	63	21	32.8	10	∗ Strong day-light.
17	17	36	111	47	15.3	10	Moon and day-light.
17	25	7	108	5	23	10	
17	27	29	116	48	25	1	
17	28	32	68	23	42.2	5	∗ Twilight.
17	30	29	116	48	42	1	
17	33	29	116	48	52	1	Day-light very strong.
17	34	36	98	38	18.5	10	Very strong twilight.
17	39	34	120	0	84	1	Most large.
17	40	41	114	52	77	1	Day-light very strong.
17	41	29	116	48	82	1	Day-light very strong.
17	43	45	105	3	80	1	Flying clouds.
17	48	0	61	18	25.6	5	Most large.
17	50	4	56	16	27.2	10	Twilight.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
17	50	7	108	5	59	1	Like the rest in this part of the heaven.
17	52	7	108	5	118	1	Many such fields just by.
17	52	17	98	43	7.6	10	
17	52	30	62	12	40	1	Most large.
17	52	32	68	19	54	1	∗ Strong day-light.
17	55	7	108	5	232	0.5	
17	55	15	106	6	112	1	Many such fields.
17	55	38	112	54	112	0.5	
17	57	30	60	28	38	1	Most large.
17	58	37	103	24	35	1	
17	58	41	118	57	64	1	
17	58	49	122	17	17	1	
17	59	1	108	8	320	0.5	
17	59	19	104	24	68	1	
18	0	13	122	11	27	1	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
18	3	49	120	42	19	1	
18	5	17	98	47	65	1	
18	6	37	90	36	9.4	10	Too soon for gaging, not having been long enough out in the dark.
18	7	4	62	14	40	1	Most large.
18	7	4	56	16	38.2	5	
18	7	37	103	25	88.0	3	
18	10	7	120	58	20	1	
18	10	52	61	8	78	1	Chiefly large.
18	11	49	104	6	170	0.5	
18	13	37	104	16	238	0.5	
18	13	52	93	11	2.0	7	
18	14	46	56	20	48	1	
18	15	28	92	42	3.4	7	
18	16	52	92	42	8.9	7	
18	18	40	92	42	13.8	7	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
18	19	37	102	34	9.5	2	
18	20	7	103	18	19	1	
18	20	46	92	42	25.8	6	
18	21	1	103	55	22	1	
18	21	12	90	41	8.6	10	
18	21	31	103	36	24	1	
18	22	4	62	7	48	1	Large and small.
18	22	4	56	16	39.6	5	
18	22	19	104	6	14	1	
18	22	37	103	45	30	1	
18	24	3	115	10	35	1	
18	24	4	109	35	35	1	Twilight.
18	24	7	102	31	30	1	
18	24	10	92	59	88	1	
18	24	43	103	39	25	1	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
18	25	37	102	34	39	1	
18	26	17	98	3	111	1	
18	26	25	103	57	60	1	
18	26	47	97	43	250	1	
18	27	1	120	58	30	1	
18	27	55	120	44	32	1	
18	28	7	102	51	13	1	Extremely small.
18	28	8	91	44	39	1	Most small.
18	28	25	103	9	20	1	Extremely small.
18	28	37	122	25	12	1	
18	29	25	103	24	20	1	Extremely small.
18	29	47	97	50	150	1	
18	29	49	121	39	24	1	
18	30	34	57	18	62	1	
18	31	10	92	42	13.7	7	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
18	31	10	108	53	74	1	Twilight.
18	31	13	103	19	112	1	All sizes.
18	31	17	97	53	188	0.5	Many more suspected.
18	31	34	62	34	76	1	∗ Large and small.
18	31	49	121	39	19.3	10	
18	33	4	108	43	88	1	Twilight.
18	33	7	103	53	146	0.5	
18	34	5	98	34	130	1	
18	34	47	71	53	78	1	
18	34	58	60	41	80	1	
18	36	34	110	12	83	1	
18	36	34	91	37	176	0.25	
18	36	47	72	28	224	0.5	∗
18	37	34	93	29	5	1	
18	38	1	104	14	118	0.5	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
18	39	40	93	52	116	0.25	
18	40	28	92	47	10	1	
18	40	47	71	48	236	0.25	*
18	41	22	91	37	156	0.25	
18	42	49	121	39	15.2	10	Very clear for this altitude.
18	43	17	72	8	368	0.25	*
18	43	33	119	21	21	1	
18	44	34	112	43	53	1	
18	44	34	60	34	84	1	All sizes.
18	47	32	91	14	328	0.25	
18	48	4	110	12	83	–	
18	50	16	60	55	136	0.5	Many of them small.
18	51	4	57	26	84	1	
18	51	32	108	26	36.8	5	Strong twilight.
18	52	49	115	30	26.2	5	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
18	54	4	57	18	93	1	
18	54	8	91	14	328	0.25	
18	54	55	104	23	180	0.5	
18	55	4	108	41	80	1	
18	55	16	62	31	206	0.5	
18	59	8	91	14	328	0.25	
18	59	26	72	37	40	1	To soon for gaging.
19	1	2	71	40	75	1	
19	1	34	56	47	127	1	Moonlight.
19	2	29	74	53	204	0.25	∗ Twilight.
19	2	37	103	16	160	0.5	
19	2	49	121	39	14.1	10	
19	3	34	55	56	146	0.5	☽
19	6	34	61	8	196	0.5	And many small besides.
19	7	34	56	56	130	0.5	☽

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
19	7	52	57	59	116	0.5	
19	8	38	92	8	120	0.5	
19	9	37	109	1	60	1	
19	9	40	56	51	130	1	☽
19	12	59	75	21	58	1	✻
19	13	50	59	59	256	0.25	
19	13	52	59	29	158	0.5	
19	14	2	72	15	60	1	✻
19	14	4	61	21	279	0 $\overline{3}$	Too crowded for accuracy.
19	14	55	103	36	64	1	Changeable focus.
19	15	40	55	26	160	1	☽ bright.
19	16	50	60	43	296	0.25	
19	16	59	73	23	56	1	✻
19	17	44	108	12	50	1	
19	18	23	78	9	196	0.25	✻

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
19	18	28	61	21	279	0 $\overline{3}$	
19	19	52	57	14	180	0.5	
19	19	56	108	12	55	1	
19	20	51	60	55	384	0.25	
19	21	1	78	47	472	0.25	∗
19	21	34	55	17	208	0.5	☽ bright.
19	22	27	62	29	320	0.25	
19	24	36	56	49	224	0.25	
19	24	49	104	24	36	1	Changeable focus.
19	24	50	60	43	296	0.25	
19	24	53	113	51	18.3	10	
19	25	4	57	9	190	0.5	☽ bright.
19	25	16	64	18	280	0.25	
19	25	22	59	36	340	0.25	
19	25	37	103	50	55	1	Changeable focus.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
19	27	36	72	34	424	0.25	∗ Too small and too crowded to be certain of the number.
19	27	44	61	8	240	0.5	
19	28	1	103	30	45	1	Changeable focus.
19	28	6	56	49	288	0.25	
19	28	52	59	26	344	0.25	
19	28	52	56	47	186	0.5	☽ very bright.
19	29	46	65	10	34	1	
19	30	36	74	33	588	0.25	∗
19	30	36	54	53	312	0.25	
19	31	33	92	34	62.2	5	∗
19	32	9	109	44	23.8	10	
19	32	15	62	35	296	0.25	
19	33	4	55	34	212	0.5	☽
19	33	7	103	12	50	1	Changeable focus.
19	33	14	61	8	240	0 $\overline{3}$	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
19	33	20	58	59	232	0.25	
19	34	51	115	44	14.1	10	
19	35	34	63	19	256	0.25	
19	36	6	54	57	384	0.25	
19	36	37	102	31	68	1	
19	36	50	60	35	296	0.25	
19	40	33	63	0	296	0.25	
19	40	46	59	12	192	0.25	
19	40	48	74	33	588	0.25	∗
19	42	33	73	14	352	0.25	∗
19	43	30	57	23	130	0.5	☽
19	43	56	64	27	124	0.5	Most large.
19	45	36	77	58	140	0.5	∗ Faint ☽.
19	45	37	103	3	50	1	
19	46	21	73	14	352	0.25	∗

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
19	46	51	115	44	12.8	10	Strong twilight.
19	47	8	60	35	312	0.25	
19	47	18	109	46	20.9	10	
19	47	22	57	38	312	0.25	Very unequally scattered.
19	49	6	57	13	268	0.25	
19	49	48	56	51	120	0.5	☽
19	50	5	92	39	39.2	5	∗ Most small.
19	51	37	62	37	51	1	
19	52	0	57	15	220	0.5	☽
19	53	1	60	36	80	1	
19	53	28	63	40	52	0.5	
19	53	40	54	59	306	0.25	
19	53	49	121	39	7.7	10	
19	54	0	55	12	160	0.5	☽
19	54	12	78	3	120	0.5	∗ Faint ☽.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
19	54	22	59	58	136	0.25	
19	55	7	62	41	48	1	
19	56	19	60	44	112	0.5	
19	56	22	57	17	192	0.25	
19	57	19	62	34	45	1	
19	57	40	58	29	104	0.5	
19	59	49	62	37	41	1	
20	0	21	79	3	56	1	∗ Strong ☽.
20	0	24	55	12	184	0.5	☽
20	0	25	60	33	80	1	Most of the stars extremely small.
20	0	51	115	44	12.2	10	Twilight.
20	1	39	79	34	68	1	∗ Strong ☽.
20	5	26	56	34	46	1	☽
20	5	27	72	56	280	0.25	
20	6	23	107	27	22.6	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
20	6	43	62	32	75	1	Many small.
20	8	26	56	27	47.4	5	☽
20	8	27	72	56	280	0.25	
20	8	58	103	37	38	1	
20	9	6	109	40	24.2	5	
20	9	52	102	48	31	1	
20	12	22	58	14	76	0.5	
20	17	20	76	12	184	0.25	Some twilight.
20	18	51	115	44	10.6	10	Twilight.
20	20	58	61	27	88	1	
10	21	36	71	28	104	0.5	Hazy.
20	22	56	56	27	66	1	☽
20	22	58	103	26	20	1	
20	24	51	115	44	9.3	10	Twilight.
20	25	58	103	26	22.8	10	Changeable focus.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
20	25	59	67	27	248	0.25	
20	26	1	92	44	30.8	5	✻
20	26	46	109	37	16.7	10	Not clear.
20	26	49	121	39	7.7	10	A little hazy.
20	27	33	96	7	39	1	Most small.
20	34	51	115	44	9.5	10	☽
20	35	53	61	20	142	0.5	
20	37	18	58	28	108	0.5	
20	37	34	97	6	26.6	10	✻
20	38	1	92	44	28.2	5	✻
20	39	42	66	37	78	0.5	
20	40	22	56	21	192	0.25	
20	41	11	67	54	108	0.5	
20	41	56	74	33	116	0.5	
20	42	59	62	14	112	0.5	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
20	43	1	70	29	76	1	
20	43	30	54	47	260	0.25	Most of the stars of the same size.
20	44	59	70	6	80	1	
20	47	13	60	46	120	0.5	
20	49	1	92	44	27.0	5	∗
20	49	10	57	11	248	0.25	Most of a size.
20	50	59	103	26	17.2	3	
20	51	23	68	30	70	1	
20	53	29	103	26	17.4	5	
20	54	1	107	47	10.3	10	
20	56	59	103	26	14.9	10	Most extremely small.
20	57	55	61	25	64	1	Twilight.
20	59	1	92	44	21.4	5	∗
21	1	6	96	43	40	1	∗ Most small.
21	3	29	66	39	80	0.5	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
21	3	53	73	9	55	1	
21	6	13	69	23	40	1	A little hazy.
21	6	55	103	32	11.1	10	
21	7	49	109	45	12.8	10	
21	7	59	64	58	110	0.5	
21	9	25	61	36	75	1	Strong twilight.
21	10	13	60	39	70	1	Strong twilight.
21	11	17	73	18	50	1	
21	11	42	96	13	25	1	*
21	12	1	92	44	16.4	5	
21	15	3	109	56	15.3	10	
21	16	43	59	7	76	0.5	
21	18	54	57	20	50	1	
21	20	18	96	43	24	1	*
21	21	0	107	49	8.1	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
21	22	14	76	33	30.0	5	
21	25	31	92	44	8.0	5	
21	29	12	83	11	21.6	5	
21	30	58	78	57	18.9	10	
21	32	10	57	14	25	1	
21	33	1	92	44	15.4	5	Strong twilight.
21	34	55	97	17	13.6	10	*
21	36	38	65	55	42	1	
21	38	20	65	38	60	1	
21	39	55	96	17	18	1	*
21	41	52	58	42	44	1	
21	43	22	109	55	11.5	10	*
21	45	4	59	39	52	1	
21	48	22	59	30	29	1	
21	51	52	58	56	61	1	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
21	51	55	97	17	11.5	10	⁎
21	54	22	109	55	12.8	10	⁎
21	57	49	59	37	60	0.5	
21	58	4	75	7	33	1	
21	58	19	59	6	40	0.5	
21	58	43	58	34	32.6	5	☽
21	58	49	58	20	34	1	
22	2	25	60	9	42.6	5	
22	2	52	109	55	7.4	10	⁎
22	3	56	71	48	25.1	10	
22	7	22	109	55	8.9	10	⁎
22	10	28	75	2	26	1	
22	11	32	97	14	10.7	10	⁎ Twilight.
22	11	35	65	48	26.6	5	
22	18	32	97	14	9.1	10	⁎ Twilight.

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
22	20	35	109	58	8.3	10	✻
22	20	55	78	54	11.7	10	Bright ☽.
22	27	41	95	4	8.1	10	
22	30	35	109	58	5.0	10	✻
22	31	28	73	59	17.3	10	
22	33	6	76	52	16.5	10	
22	34	40	61	56	20.1	10	
21	35	35	109	58	7.1	10	✻
22	36	49	71	57	18.5	10	
22	39	41	82	5	19	1	
22	40	5	65	48	21.3	10	
22	43	55	60	9	26.7	10	Faint ☽.
22	45	3	80	47	13.2	10	
22	45	30	58	38	17.2	10	☽
22	48	49	71	57	13.4	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
22	52	9	78	43	8.2	10	☽
22	52	41	95	4	8.9	10	
22	55	40	71	54	11.6	10	
22	56	55	67	53	12.1	10	
22	58	19	78	42	9.2	10	☽
23	0	27	113	12	4.4	10	
23	0	30	58	38	18.7	10	☽
23	2	59	65	50	21.3	10	
23	5	35	109	58	7.3	10	☽
23	8	52	95	1	7.5	10	Most extremely small.
23	10	4	64	55	26	1	
23	11	40	61	48	21.1	10	
23	12	40	71	54	11.9	10	
23	17	50	81	0	9.7	10	
23	23	58	69	48	12.1	10	

Table 1 continued on the next page

Right Ascension			Polar Distance		Stars	Fields	Memorandums
H.	M.	S.	D.	M.			
23	25	32	113	12	3.1	10	*
23	32	2	69	51	9.5	10	
23	33	20	79	45	10	1	
23	43	2	69	51	10.9	10	
23	44	47	45	24	50	1	
23	46	52	113	17	4.2	10	*
23	46	55	65	36	15.3	10	
23	59	21	87	10	5.6	10	
23	59	56	95	4	7.8	10	

7 Problem

The stars being supposed to be nearly equally scattered, and their number, in a field of view of a known angular diameter, being given, to determine the length of the visual ray.

Here, the arrangement of the stars not being fixed upon, we must endeavour to find which way they may be placed so as to fill a given space most equally. Suppose a rectangular cone cut into frusta by many equidistant planes perpendicular to the axis; then, if one star be placed at the vertex, and another in the axis at the first intersection, fix stars may be set around it so as to be equally distant from one another and from the central star. These positions being carried on in the same manner, we shall have every star within the cone surrounded by eight others, at an equal distance from that star taken as a centre. Figure 3 (tab. VIII.) (*Fig. 1*) contains four sections of such a cone distinguished by alternate shades, which will be Sufficient to explain what sort of arrangement I would point out.
The series of the number of stars contained in the several sections will be $1, 7, 19, 37, 61, 91,$ &*c* which continued to n terms, the sum of it, by the differential method, will be

$$na + n \cdot \frac{n-1}{2}\,\mathrm{d}n + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}\,\mathrm{d}^2 n + \cdots \qquad (1)$$

where a is the first term $\mathrm{d}n$, $\mathrm{d}^2 n$, $\mathrm{d}^3 n$, etc. the 1st, 2d, and 3d differences. Then, since $a = 1$, $\mathrm{d}n = 6$, $\mathrm{d}^2 n = 6$, $\mathrm{d}^3 n = 0$, the sum of the series will be n^3. Let S be the given number of stars; ℓ, the diameter of the base of the field of view; and B, the diameter of the base of the great rectangular cone; and, by trigonometry, we shall have

$$B = \frac{\text{Radius}}{\tan\left(\frac{1}{2} \cdot \text{ field}\right)}\,. \qquad (2)$$

Now, since the field of view of a telescope is a cone, we shall have its solidity to that of the great cone of stars, formed by the above construction as the square of the diameter of the base of the field of view, to the square of the diameter of the base of the great cone, the height of both being the same; and the stars in each cone being in the ratio of the solidity, as being equally scattered[2], we have

$$n = \sqrt[3]{B^2 \cdot S}\,. \tag{3}$$

And the length of the visual ray is equal to $n - 1$, which was to be determined.

[2]We ought to remark that the periphery and base of the cone of the field of view, in gaging, would in all probability seldom fall exactly on such stars as would produce a perfect equality of situation between the stars contained in the small and the great cone; and that, consequently, the solution of this problem, where we suppose the stars of one cone to be to those of the other in the ratio of the solidity on account of their being equally scattered, will not be strictly true. But it should be remembered, that in small numbers, where the different terminations of the fields would most affect this solution, the stars in view have always been ascertained from gages that were often repeated, and each of which consisted of no less than ten fields successively taken, so that the different deviations at the periphery and base of the cone would certainly compensate each other sufficiently for the purpose of this calculation. And that, on the other hand, in high gages, which could not have the advantage of being so often repeated, these deviations would bear a much smaller proportion to the great number of stars in a field of view, and therefore, on this account, such gages may very justly be admitted in a solution where practical truth rather than mathematical precision is the end we have in view. It is moreover not to be supposed that we imagine the stars to be actually arranged in this regular manner, and, returning therefore to our general hypothesis of their being equally scattered, any one field of view promiscuously taken may, in this general sense, be supposed to contain a due proportion of them; so that the principle on which this solution is founded may therefore be said to be even more rigorously true than we have occasion to insist upon in an argument of this kind.

8 The same otherwise

If a different arrangement of the stars should be selected, such as that in Figure 3 (*Fig. 2*), where one star is at the vertex of a cone; three in the circumference of the first section, at an equal distance from the vertex and from each other; fix in the circumference of the next section, with one in the axis or centre; and so on, always placing three stars in a lower section in such a manner as to form an equilateral pyramid with one above them: then we shall have every star, which is sufficiently within the cone, surrounded by twelve others at an equal distance from the central star and from each other. And by the differential method, the sum of the two series equally continued, into which this cone may be resolved, will be

$$2n^3 + \frac{3}{2}n^2 + \frac{1}{2}n\,;$$

where n stands for the number of terms in each series. To find the angle which a line $v \cdot x$, passing from the vertex v over the stars v, n, b, l, &c. to x, at the outside of the cone, makes with the axis; we have, by construction, $v \cdot s$ in Figure 3 (*Fig. 3*) representing the planes of the first and second sections, which equal $2 \cdot \cos(30) = \phi$, to the radius $p \cdot s$, of the first section that equal 1. Hence it will be

$$\sqrt{\phi^2 - 1} = v \cdot p = \frac{1}{2} \cdot v \cdot m$$

or

$$v \cdot m = 2 \cdot \sqrt{\phi^2 - 1}\,.$$

By trigonometry we have

$$\frac{R \cdot \phi}{2 \cdot \sqrt{\phi^2 - 1}} = T\,.$$

Where T is the tangent of the required angle to the radius R; and putting t = tangent of half the given field of view, it will be

$$\frac{T}{t} = B\,,$$

the base of the cone.[3] And

$$\frac{\sqrt{\phi^2 - 1}}{\phi} = d$$

will be an expression for $v \cdot p$ in terms of $v \cdot s$, which is the mutual distance of the scattered stars. Then having

$$\frac{B^2 \cdot S}{2} = n^3 + \frac{3}{4} \cdot n^2 + \frac{1}{4} \cdot n\,,$$

we may find n; whence

$$2 \cdot n \cdot d - d\,,$$

the visual ray will be obtained.
The result of this arrangement gives a shorter ray than that of the former; but since the difference is not so considerable as very materially to affect the conclusions, I shall, on account of the greater convenience, make use of the first.

[3] In finding this angle we have supposed the cone to be generated by a revolving rectangular triangle of which the line $v \cdot x$, Figure 3 (*Fig. 2*), is the hypotenuse; but the stars in the second series will occasion the cone to be contained under a waving surface, wherefore the above supposition of the generation of the cone is not strictly true; but then these waves are so inconsiderable, that, for the present purpose, they may safely be neglected in this calculation.

9 We inhabit the planet of a star belonging to a Compound Nebula of the third form

I shall now proceed to shew that the stupendous sidereal system we inhabit, this extensive stratum and its secondary branch, consisting of many millions of stars, is, in all probability, a *detached Nebula*. In order to go upon grounds that seem to me to be capable of great certainty, they being no less than an actual survey of the boundaries of our sidereal system, which I have plainly perceived, as far as I have yet gone round it, everywhere terminated, and in most places very narrowly too, it will be proper to shew the length of my founding line, if I may so call it, that it may appear whether it was sufficiently long for the purpose.

In the most crowded part of the milky way I have had fields of view that contained no less than 588 stars[4] and these were continued for many minutes, so that in one quarter of an hour's time there passed no less than 116 000 stars through the field of view of my telescope[5]. Now, if we compute the length of the visual ray by putting $S = 588$, and the diameter of the field of view fifteen minutes, we shall find $n = \sqrt[3]{B^2 \cdot S} = 498$; so that it appears the length of what I have called my founding line, or $n - 1$, was probably not less than 497 times the distance of Sirius from the sun. The same gage calculated by the second arrangement of stars gives:

[4] See the table of Gages on page 42.

[5] The breadth of my sweep was $2^\circ\, 26'$, to which must be added $15'$ for two semi-diameters of the field. Then, putting $161 = a$, the number of fields in 15 minutes of time; $0.7854 = b$, the proportion of a circle to 1, its circumscribed square; $\phi =$ sin of $74^\circ\, 22'$, the polar distance of the middle of the sweep reduced to the present time; and $588 = S$, the number of stars in a field of view, we have $\frac{a \cdot \phi \cdot S}{b} = 116\,076$ stars.

$$\sqrt{\phi^2 - 1} = 1.414\,21$$
$$\frac{R \cdot \phi}{2 \cdot \sqrt{\phi^2 - 1}} = \tan \text{ of } 31^\circ\, 28'\, 55'',\, 77;$$
$$\frac{T}{t} = B = 280.69\,;$$
$$\frac{\sqrt{\phi^2 - 1}}{\phi} = d = 0.816\,49\,;$$
$$\frac{B^2 \cdot S}{2} = 23\,163\,409.7 = n^3 + \frac{3}{4}n^2 + \frac{1}{4}n\,,$$

where $n = 284.8$ nearly; and

$$2 \cdot d \cdot n - 1 = 464\,,$$

the visual ray.

It may seem inaccurate that we should found an argument on the stars being equally scattered, when in all probability there may not be two of them in the heavens, whose mutual distance shall be equal to that of any other two given stars; but it should be considered, that when we take all the stars collectively there will be a mean distance, which may be assumed as the general one; and an argument founded on such a supposition will have in its favour the greatest probability of not being far short of truth. What will render the supposition of an equal distribution of the stars, with regard to the gages, still less exposed to objections is, that whenever the stars happened either to be uncommonly crowded or deficient in number, so as very suddenly to pass over from one extreme to the other, the gages were reduced to other forms, such as the border-gage, the distance-gage,& c. which terms, and the use of such gages, I shall hereafter find an opportunity of explaining. And none of those kinds of gages have been admitted in this table, which consists only of such as have been taken in places where the stars apparently seemed to be, in general, pretty evenly scattered; and to increase and decrease in number by a certain gradual progression. Nor has any part of the heavens containing a cluster of stars been put in the gages; and here I must observe, that the difference between a crowded place and a cluster may easily be perceived by the arrangement as well

as the size and mutual distance of the stars: for in a cluster they are generally not only resembling each other pretty nearly in size, but a certain uniformity of distance also takes place; they are more and more accumulated towards the centre, and put on all the appearances which we should naturally expect from a number of them collected into a group at a certain distance from us. On the other hand, the rich parts of the Milky Way, as well as those in the distant broad part of the stratum, consist of a mixture of stars of all possible sizes that are seemingly placed without any particular apparent order. Perhaps we might recollect that a greater condensation towards the centre of our system than towards the borders of it should be taken into consideration; but, with a nebula of the third form, containing such various and extensive combinations, as I have found to take place in ours, this circumstance, which in one of the first form would be of considerable moment, may, I think, be safely neglected. However, I would not be understood to lay a greater stress on these and the following calculations than the principles on which they are founded will permit; and if hereafter we shall find a reason, from experience and observation, to believe that there are parts of our system where the stars are not scattered in the manner here supposed, we ought then to make proper exceptions.

But to return: if some other high gage be selected from the table, such as 472 or 344, the length of the visual ray will be found 461 and 415. And although, in consequence of what has been said, a certain degree of doubt may be less about the arrangement and scattering of the stars, yet when I recollect, that in those parts of the milky way where these high gages were taken, the stars were neither so small, nor so crowded, as they must have been on a supposition of a much farther continuance of them, when certainly a milky or nebulous appearance must have come on, I need not fear to have overrated the extent of my visual ray. And indeed everything that can be said to shorten it will only contract the limits of our nebula, as it has in most places been of sufficient length to go far beyond the bounds of it. Thus, in the sides of the stratum opposite to our situation in it, where the gages often run below 5, our nebula cannot extend to 100 times the distance of Sirius; and the same telescope, which could shew 588 stars in a field of view of 15 minutes, must certainly have presented me also with the stars in these situations as well as the former,

had they been there. If we should answer this by observing that they might be at too great a distance to be perceived, it will be allowing that there must at least be a vacancy mounting to the length of a visual ray not short of 400 times the distance of Sirius; and this is amply sufficient to make our nebula a detached one. It is true, that it would not be consistent confidently to affirm that we were on an island unless we had actually found ourselves everywhere bounded by the ocean, and therefore I shall go no farther than the gages will authorise; but considering the little depth of the stratum in all those places which have been actually gaged, to which must be added all the intermediate parts that have been viewed and found to be much like the rest, there is but little room to expect a connection between our nebula and any of the neighbouring ones. I ought also to add, that a telescope with a much larger aperture than my present one, grasping together a greater quantity of light, and thereby enabling us to see farther into space, will be the surest means of completing and establishing the arguments that have been used: for if our nebula is not absolutely a detached one, I am firmly persuaded that an instrument may be made large enough to discover the places where the stars continue onwards. A very bright milky nebulosity must there undoubtedly come on, since the stars in a field of view will increase in the ratio of n^3, greater than that of the cube of the visual ray. Thus, if 588 stars in a given field of view are to be seen by a ray of 497 times the distance of Sirius; when this is lengthened to 1000, which is but little more than double the former, the number of stars in the same field of view will be no less than 4774: for when the visual ray r is given, the number S of stars will be

$$S = \frac{n^3}{B^2} \ :$$

where $n = r + 1$; and a telescope with a three-fold power of extending into space, or with a ray of 1500, which, I think, may easily be constructed, will give us 16 096 stars. Now, these would not be so close but that a good power applied to such an instrument might easily distinguish them; for they need not, if arranged in regular squares, approach nearer to each other than $6'' 27$; but what would produce the milky nebulosity, which I

have mentioned is the numberless stars beyond them, which in one respect the visual ray might also be said to reach. To make this appear, we must return to the naked eye, which, as we have before estimated, can only see the stars of the seventh magnitude so as to distinguish them; but it is nevertheless very evident that the united cluster of millions of stars, such as I suppose the nebula in Andromeda to be, will reach our sight in the shape of a very small, faint nebulosity; since the nebula of which I speak may easily be seen in a fine evening. In the same manner my present telescope, as I have argued, has not only a visual ray that will reach the stars at 497 times the distance of Sirius so as to distinguish them (and probably much farther), but also a power of shewing the united lustre of the accumulated stars that compose a milky nebulosity, at a distance far exceeding the former limits; so that from these considerations it appears again highly probable that my present telescope, not shewing such a nebulosity in the milky way, goes already far beyond its extent: and consequently, much more would an instrument, such as I have mentioned, remove all doubt on the subject, both by shewing the stars in the continuation of the stratum, and by exposing a very strong milky nebulosity beyond them, that could no longer be mistaken for the dark ground of the heavens.

To these arguments, which rest on the firm basis of a series of observation, we may add the following considerations drawn from analogy. Among the great number of nebulæ which I have now already seen, amounting to more than 900, there are many which in all probability are equal extensive with that which we inhabit; and yet they are all separated from each other by very considerable intervals. Some indeed there are that seem to be double and treble: and though with most of these it may be, that they are at a very great distance from each other, yet we allow that some such conjunctions really are to be found; nor is this what we mean to exclude. But then these compound or double nebulæ, which are those of the third and fourth forms, still make a detached link in the great chain. It is also to be supposed, that there may still be some thinly scattered solitary stars between the large interfaces of nebulæ, which, being situated so as to be nearly equally attracted by the several clusters when they were forming, remain unassociated. And though we cannot expect to see these stars, on account of their vast distance, yet we may

well presume, that their number cannot be very considerable in comparison to those that are already drawn into systems; which conjecture is also abundantly confirmed in situations where the nebulæ are near enough to have their stars visible; for they are ail insulated, and generally to be seen upon a very clear and pure ground, without any star near them that might be supposed to belong to them. And though I have often seen them in beds of stars, yet from the size of these latter we may be certain, that they were much nearer to us than those nebulæ, and belonged undoubtedly to our own system.

10 Use of the Gages

A delineation of our nebula, by an application of the gages in the manner which has been proposed to be done in my former paper, may now be attempted, and the following Table 2 is calculated for this purpose. It gives us the length of the visual ray for any number of stars in the field of view contained in the third column of the foregoing table of gages from $\frac{1}{10}$ to 100 000. If the number required is not to be found in the first column of this table, a proportional mean may be taken between the two nearest rays in the second column, without any material error, except in the few last numbers. The calculations of resolvable and milky nebulosity, at the end of the table, are founded, the first, on a supposition of the stars being so crowded as to have only a square second of space allowed them; the next assigning them only half a second square. However, we should consider that in all probability a very different accumulation of stars may take place in different Nebulæ; by which means some of them may assume the milky appearance, though not near so far removed from us; while clusters of stars also may become resolvable nebulæ from the same cause. The distinctness of the instrument is here also concerned; and as telescopes with large apertures are not easily brought to a good figure, nebulous appearances of both sorts may probably come on much before the distance annexed to them in the table.

Stars in the field	Visual ray	Stars	Ray	Stars	Ray	Stars	Ray	Stars	Ray
		31	186	71	245	210	352	700	527
0.1	27	32	188	72	246	220	358	800	551
0.2	34	33	190	73	247	230	363	900	573
0.3	39	34	192	74	249	240	368	1000	593
0.4	43	35	193	75	250	250	374	10 000	1280
0.5	46	36	195	76	251	260	378	100 000	2758
0.6	49	37	197	77	252	270	383	–	–
0.7	52	38	199	78	253	280	388	–	–
0.8	54	39	201	79	254	290	393	–	–
0.9	56	40	202	80	255	300	397	–	–
1	58	41	204	81	256	310	401	–	–
2	74	42	206	82	257	320	406	–	–
3	85	43	207	83	258	330	410	–	–
4	93	44	209	84	259	340	414	–	–
5	101	45	210	85	260	350	418	–	–
6	107	46	212	86	261	360	422	–	–
7	113	47	214	87	262	370	426	–	–
8	118	48	215	88	263	380	430	–	–
9	123	49	217	89	264	390	433	–	–
10	127	50	218	90	265	400	437	–	–
11	131	51	219	91	266	410	441	–	–
12	135	52	221	92	267	420	444	–	–
13	139	53	222	93	268	430	448	–	–
14	142	54	224	94	269	440	451	–	–
15	146	55	225	95	270	450	455	–	–
16	149	56	226	96	271	460	458	–	–
17	152	57	228	97	272	470	461	–	–
18	155	58	229	98	273	480	464	–	–
19	158	59	230	99	274	490	468	–	–
20	160	60	232	100	275	500	471	–	–
21	163	61	233	110	284	510	474	–	–
22	166	62	234	120	291	520	477	–	–
23	168	63	236	130	300	530	480	–	–
24	170	64	237	140	308	540	483	–	–
25	173	65	238	150	315	550	486	–	–
26	175	66	239	160	322	560	489	–	–
27	177	67	240	170	328	570	492	–	–
28	180	68	242	180	335	580	495	–	–
29	182	69	243	190	341	590	498	–	–
30	184	70	244	200	347	600	500	–	–

Table 2: Calculated delineation values of nebulæ

11 Section of our Sidereal System

By taking out of this table the visual rays which answer to the gages, and applying lines proportional to them around a point, according to their respective right ascensions and north polar distances, we may delineate a solid by means of the ends of these lines, which will give us so many points in its surface; I shall, however, content myself at present with a section only. I have taken one which passes through the poles of our system, and is at rectangles to the conjunction of the branches which I have called its length. The name of poles seemed to me not improperly applied to those points which are 90 degrees distant from a circle passing along the milky way, and the north pole is here assumed to be situated in R.A. 186° and P.D. 58°. The section represented in Figure 3 (*Fig. 4*) is one which makes an angle of 35 degrees with our equator, crossing it in 124 ½ and 304 ½ degrees. A celestial globe, adjusted to the latitude of 55° north, and having σ Ceti near the meridian, will have the plane of this section pointed out by the horizon, and the gages which have been used in this delineation are those which in Table 1 are marked by asterisk. When the visual rays answering to them are taken out of the second table, they must be projected on the plane of the horizon of the latitude which has been pointed out; and this may be done accurately enough for the present purpose by a globe adjusted as above directed; for as gages, exactly in the plane of the section, were often wanting, I have used many at some small distance above and below the same, for the sake of obtaining more delineating points; and in the figure the stars at the borders which are larger than the rest are those pointed out by the gages. The intermediate parts are filled up by smaller stars arranged in straight lines between the gaged ones. The delineating points, though pretty numerous, are not so close as we might wish; it is however to be hoped that in some future time this branch of astronomy will become more cultivated, so that we may have gages for every quarter of a degree of the heavens at least, and these often repeated in the most favourable circumstances. And whenever that shall be the case, the delineations may then be repeated with all the accuracy that long experience may enable us to introduce; for, this subject being so new, I look upon what is here given partly as only an example to illustrate the spirit of

the method. From Figure 3 (*Fig. 4*) however, which I hope is not a very inaccurate one, we may see that our nebula, as we observed before, is of the third form; that is: *A very extensive, branching, compound Congeries of many millions of stars*; which most probably owes its origin to many remarkably large as well as pretty closely scattered small stars, that may have drawn together the rest. Now, to have some idea of the wonderful extent of this system, I must observe that this section of it is drawn upon a scale where the distance of Sirius is no more than the 80th part of an inch; so that probably all the stars, which in the finest nights we are able to distinguish with the naked eye, may be comprehended within a sphere, drawn round the large star near the middle, representing our situation in the nebula, of less than half a quarter of an inch radius.

12 The Origin of Nebulous Strata

If it were possible to distinguish between the parts of an indefinitely extended whole, the nebula we inhabit might be said to be one that has fewer marks of profound antiquity upon it than the rest. To explain this idea perhaps more clearly, we should recollect that the condensation of clusters of stars has been ascribed to a gradual approach; and whoever reflects on the numbers of ages that must have passed before some of the clusters, that will be found in my intended catalogue of them, could be so far condensed as we find them at present, will not wonder if I ascribe a certain air of youth and vigour to many very regularly scattered regions of our sidereal stratum. There are moreover many places in it where there is the greatest reason to believe that the stars, if we may judge from appearances, are now drawing towards various secondary centres, and will in time separate into different clusters, so as to occasion many sub-divisions. Hence, we may surmise that when a nebulous stratum consists chiefly of nebulæ of the first and second form, it probably owes its origin to what may be called the decay of a great compound nebula of the third form; and that the subdivisions, which happened to it in length of time, occasioned all the small nebulæ which sprung from it to lie in a certain range, according as they were detached from the primary one. In like

manner our system, after numbers of ages, may very possibly become divided so as to give rise to a stratum of two or three hundred nebulæ; for it would not be difficult to point out so many beginning or gathering clusters in it[6]. This view of the present subject throws a considerable light upon the appearance of that remarkable collection of many hundreds of nebulæ which are to be seen in what I have called the nebulous stratum of Coma Berenices. It appears from the extended and branching figure of our nebula, that there is room for the decomposed small nebulæ of a large, reduced, former great one to approach nearer to us in the sides than in other parts. Nay, possibly, there might originally be another very large joining branch, which in time became separated by the condensation of the stars; and this may be the reason of the little remaining breadth of our system in that very place: for the nebulæ of the stratum of the Coma are brightest and most crowded just opposite our situation, or in the pole of our system. As soon as this idea was suggested, I tried also the opposite pole, where accordingly I have met with a great number of nebulæ, though under a much more scattered form.

13 An Opening in the Heavens

Some parts of our system indeed seem already to have sustained greater ravages of time than others, if this way of expressing may be allowed; for instance, in the body the Scorpion is an opening, or hole, which is probably owing this cause. I found it while I was gaging in the parallel from 112 to 114 degrees of north polar distance. As I approached the milky way, the gages had been gradually running up from 9.7 to 17.1; when, all of a sudden, they fell down to nothing, a very few pretty large stars excepted, which made them shew 0.5, 0.7, 1.1, 1.4, 1.8; after which they again rose to 4.7, 13.5, 20.3, and soon after to 41.1. This opening is at least 4 degrees broad, but its

[6]Mr. Michell has also considered the stars as gathered together into groups, [1], which idea agrees with the sub-division of our great system here pointed out. He founds an elegant proof of this on the computation of probabilities, and mentions the Pleiades, the Præsepe Cancri, and the nebula (or cluster of stars) in the hilt of Perseus's sword, as instance.

height I have not yet ascertained. It is remarkable, that the 80 *Nebuleuse sans etoiles* of the *Connoissance des Temps*, which is one of the richest and most compressed clusters of small stars I remember to have seen, is situated just on the western border of it, and would almost authorize a suspicion that the stars, of which it is composed, were collected from that place, and had left the vacancy. What adds not a little to this surmise is, that the same phænomenon is once more repeated with the fourth cluster of stars of the *Connoissance des Temps*; which is also on the western border of another vacancy, and has moreover a small, miniature cluster, or easily resolvable nebula of about $2^1/_2$ minutes in diameter, north following it, at no very great distance.

14 Phænomena at the Poles of our Nebula

I ought to observe, that there is a remarkable purity or clearness in the heavens when we look out of our stratum at the sides; that is, towards Leo, Virgo, and Coma Berenices, on one hand, and towards Cetus on the other; whereas the ground of the heavens becomes troubled as we approach towards the length or height of it. It was a good while before I could trace the cause of these phænomena; but since I have been acquainted with the shape of our system, it is plain that these troubled appearances, when we approach to the sides, are easily to be explained by ascribing them to some of the distant, straggling stars, that yield hardly light enough to be distinguished. And I have, indeed, often experienced this to be actually the cause, by examining these troubled spots for a long while together, when, at last, I generally perceived the stars which occasioned them. But when we look towards the poles of our system where the visual ray does not graze along the side, the straggling stars of course will be very few in number; and therefore the ground of the heavens will assume that purity which I have always observed to take place in those regions.

15 Enumeration of very compound Nebulæ or Milky-Ways

As we are used to call the appearance of the heavens, where it is surrounded with a bright zone, the Milky-Way, it may not be amiss to point out some other very remarkable Nebulæ which cannot well be less, but are probably much larger than our own system; and, being also extended, the inhabitants of the planets that attend the stars which compose them must likewise perceive the same phænomena. For which reason they may also be called milky-ways by way of distinction.

My opinion of their size is grounded on the following observations. There are many round nebulæ, of the first form, of about five or six minutes in diameter, the stars of which I can see very distinctly; and on comparing them with the visual ray calculated from some of my long gages, I suppose, by the appearance of the small stars in those gages, that the centres of these round nebulæ may be 600 times the distance of Sirius from us.

In estimating the distance of such clusters I consulted rather the comparatively apparent size of the stars than their mutual distance; for the condensation in these clusters being probably much greater than in our own system, if we were to overlook this circumstance and calculate by their apparent compression, where, in about six minutes diameter, there are perhaps ten or more stars in the line of measures, we should find, that on the supposition of an equal scattering of the stars throughout all nebulæ, the distance of the centre of such a cluster from us could not be less than 6000 times the distance of Sirius. And, perhaps, in putting it, by the apparent size of the stars, at 600 only, I may have considerably underrated it; but my argument, if that should be the case, will be so much the stronger. Now to proceed...

Some of these round nebulæ have others near them, perfectly similar in form, colour, and the distribution of stars, but of only half the diameter; and the stars in them seem to be doubly crowded, at only at about half the distance from each other: they are indeed too small as not to be visible without the utmost attention. I suppose these miniature nebulæ to be at double the distance of the first. An instance, equally remarkable and

instructive, is a case where, in the neighbourhood of two such nebulæ as have been mentioned, I met with a third, similar, resolvable, but much smaller and fainter nebula. The stars of it are no longer to be perceived; but a resemblance of colour with the former two, and its diminished size and light, may well permit us to place it at full twice the distance of the second, or about four or five times that of the first. And yet the nebulosity is not of the milky kind; nor is it so much as difficultly resolvable, or colourless. Now, in a few of the extended nebulæ, the light changes gradually so as from the resolvable to approach to the milky kind; which appears to me an indication that the milky light of nebulæ is owing to their much greater distance. A nebula, therefore, whose light is perfectly milky, cannot well be supposed to be at less than six or eight thousand times the distance of Sirius; and though the numbers here assumed are not to be taken otherwise than as very coarse estimates, yet an extended nebula, which in an oblique situation, where it is possibly fore-shortened by one-half, two-third, or three-fourths of its length, subtends a degree or more in diameter, cannot be otherwise than of a wonderful magnitude, and may well outvie our milky-way in grandeur.

The first I shall mention is a milky Ray of more than a degree in length. It takes k (Fl. 52.) Cygni into its extent, to the north of which it is crookedly bent so as to be convex towards the following side; and the light of it is pretty intense. To the south of k it is more diffused, less bright, and loses itself with some extension in two branches, I believe; but for want of light I could not determine this circumstance. The northern half is near two minutes broad, but the southern is not sufficiently defined to ascertain its breadth.

The next is an extremely faint milky Ray, above $^3/_4$ degree long, and 8 or $10'$ broad; extended from north preceding to south following. It makes an angle of about 30 or 40 degrees with the meridian, and contains three or four places that are brighter than the rest. The stars of the Galaxy are scattered over it in the same manner as over the rest of the heavens. It follows ε Cygni 11,5 minutes in time, and is $2^\circ\,19'$ more south. The third is a branching Nebulosity of about a degree and a half in right ascension, and about $48'$ extent in polar distance. The following part of it is divided into several streams and windings, which,

after separating, meet each other again towards the south. It precedes ζ Cygni $16'$ in time, and is $1^\circ\,16'$ more north. I suppose this to be joined to the preceding one; but having observed them in different sweeps, there was no opportunity of tracing their connection.

The fourth is a faint, extended milky Ray of about $17'$ in length, and $12'$ in breadth. It is brightest and broadest middle, and the ends lose themselves. It has a small, round, very faint nebula just north of it; and also, in another place, a spot, brighter than the rest, almost detached enough to form a different nebula, but probably belonging to the great one. The Ray precedes α Trianguli $18',8$ in time, and is $55'$ more north. Another observation of the same, in a finer evening, mentions its extending much farther towards the south, and that the breadth of it probably is not less than half a degree; but being shaded away by imperceptible gradations, it is difficult exactly to assign its limits.

The fifth is a Streak of light about $27'$ long, and in the brightest part 3 or $4'$ broad. The extent is nearly in the meridian, or a little from south preceding to north following. It follows β Ceti $5',9$ in time, and is $2^\circ\,43'$ more south. The situation is so low, that it would probably appear of a much greater extent in a higher altitude.

The sixth is an extensive milky Nebulosity divided into two parts; the most north being the strongest. Its extent exceeds $15'$; the southern part is followed by a parcel of stars which supposed to be the 8th of the *Connoissance des Temps*.

The seventh is a wonderful, extensive Nebulosity of the milky kind. There are several stars visible in it, but they can have no connection with that nebulosity, and are, doubtless, belonging to our own system scattered before it. It is the 17th of the *Connoissance des Temps*.

In the list of these must also be reckoned the beautiful Nebula of Orion. Its extent is much above one degree; the eastern branch passes between two very small stars, and runs on till it meets a very bright one. Close to the four small stars, which can have no connection with the nebula, is a total blackness; and within the open part, towards the north-east, is a distinct, small, faint nebula, of an extended shape, at a distance from the border of the great one, to which it runs in a parallel direction, resembling the shoals that are seen near the coasts of some islands.

The ninth is that in the girdle of Andromeda, which is undoubtedly the nearest of all the great nebulæ; its extent above a degree and a half in length, and, in even one of the narrowest places, not less than $16'$ in breadth. The brightest part of it approaches to the resolvable nebulosity, and begins to shew a faint red colour; which, from many observations on the colour and magnitude of nebulæ, I believe to be an indication that its distance in this coloured part does not exceed 2000 times the distance of Sirius. There is a very considerable, broad, pretty faint, small nebula near it; my sister discovered it August 27, 1783, with a NEWTONian 2-feet sweeper. It shews the same faint colour with the great one, and is, no doubt, in the neighbourhood of it. It is not the 32nd of the *Connoissance des Temps*; which is a pretty large round nebula, much condensed in the middle, and south following the great one; but this is about two-thirds of a degree north preceding it, in a line parallel to β and ν Andromedæ.
To these may be added the nebula in Vulpecula; for, though its appearance is not large, it is probably a double stratum of stars of a very great extent, one end whereof is turned towards us. That it is thus situated may be surmised from its containing, in different parts, nearly all the three nebulosities; *viz.* the resolvable, the coloured but irresolvable, and a tincture of the milky kind. Now, what great length must be required to produce these effects may easily be conceived when, in all probability, our whole system, of about 800 stars in diameter, in it were seen at such a distance that one end of it might assume the resolvable nebulosity, would not, at the other end, presents us with the irresolvable, much less with the colourless and milky sort of nebulosities.

16 A Perforated Nebula, or Ring of Stars

Among the curiosities of the heavens should be placed a nebula that has a regular, concentric, dark spot in the middle, and is probably a Ring of stars. It is of an oval shape, the shorter axis being to the longer as about 83 to 100; so that, if the stars form a circle, its inclination to a line drawn from the sun to the centre of this nebula must be about 56 degrees. The light is of the resolvable kind, and in the northern side three very faint stars may be seen, as also one or two in the southern part. The vertices of the longer axis seem less bright and not as well defined as the rest. There are several small stars very near, but none that seem to belong to it. It is the 57th of the *Connoissance des Temps*. Figure 3 (*Fig. 5*) is a representation of it.

17 Planetary Nebulæ

I shall conclude this paper with an account of a few bodies, that from their singular appearance leave me almost no doubt where to class them.

The first precedes ν Aquarii $5',4$ in time, and is $1'$ more north. Its place, with regard to a small star September 7, 1782, was, Distance $8'\,13''\,51'''$; but on account of the low situation, and other unfavourable circumstances, the measure cannot be very exact. August 25, 1783, Distance $7'\,5''\,11'''$, very exact, and to my satisfaction; the light being thrown in by an opaque-microscopic illumination[7]. September 20, 1783, Position $41^\circ\,24'$ south preceding the same star; very exact, and by the same kind of illumination. October 17, 1783, Distance $6'\,55''\,7'''$; a second measure $6'\,56''\,11'''$, as exact as possible. October 23, 1783, Position $42^\circ\,57'$; a second measure $42^\circ\,45'$; single lens; power 71; opaque-microscopic illumination. November 14, 1783, Distance $7'\,4''\,35'''$. November 12, 1784, Distance $7'\,22''\,35'''$; Position $38^\circ\,39'$. Its diameter is about 10 or $15''$. I have examined it with the powers of 71, 227, 278, 460, and 932; and it follows the laws of magnifying, so that its body is no illusion of light. It is a little oval, and in the 7-feet reflector pretty well defined, but not sharp on the edges. In the 20-feet, of 18.7 inch aperture, it is much better defined, and has much of a planetary appearance, being all over of an uniform brightness, in which it differs from nebulæ: its light seems however to be of the starry nature, which suffers not nearly so much as the planetary disks are known to do, when much magnified.

The second of these bodies precedes the 13th of FLAMSTEED's Andromeda about $1'6$ in time, and is $22'$ more south. It has a round, bright, pretty well-defined planetary disk of about $12''$ diameter, and is a little elliptical. When it is viewed with a 7-feet reflector, or other inferior instruments, it is not nearly too

[7] It may be of use to explain this kind of illumination for which the NEWTONian reflector is admirably constructed. On the side opposite the eye-piece an opening is to be made in the tube, through which the light may be thrown in, so as to fall on some reflecting body, or concave perforated mirror, within the eye-piece that may throw it back upon the wires. By this means none of the direct rays can reach the eye, and those few which are reflected again from the wires do not interfere, sensibly with the faintest objects, which may thus bet seen undisturbed.

well defined as with the 20-feet. Its situation with regard to a pretty considerable star is, Distance (with a compound glass of a low power) $7'\,51''\,34'''$. Position $12^\circ\,0'$ south preceding. Diameter taken with 278, $14''\,42''$.

The third follows B (Fl. 44.) Ophiuchi $4'\,1$ in time, and is $23'$ more north. It is round, tolerably well defined, and pretty bright; its diameter is about $30''$.

The fourth follows η Sagittæ $17'\,1$ in time, and is $2'$ more north. It is perfectly round, pretty bright, and pretty well defined; about $^3/_4$ min in diameter.

The fifth follows the 21st Vulpeculæ $2'\,1$ in time, and is $1^\circ\,46'$ more north. It is exactly round, of an equal light throughout, but pretty faint, and about $1'$ in diameter.

The sixth precedes h (Fl. 39,) Cygni $8'\,1$ in time, and is $1^\circ\,26'$ more south. It is perfectly round, and of an equal light, but pretty faint; its diameter is near $1'$, and the edges are pretty well defined.

The planetary appearance of the two first is so remarkable, that we can hardly suppose them to be nebulæ; their light is so uniform, as well as vivid, the diameters so small and well defined, as to make it almost improbable they should belong to that species of bodies. On the other hand, the effect of different powers seems to be much against their light's being of a planetary nature, since it preserves its brightness nearly in the same manner as the stars do in similar trials. If we would suppose them to be single stars with large diameters, we shall find it difficult to account for their not being brighter; unless we should admit that the intrinsic light of some stars may be very much inferior to that of the generality, which however can hardly be imagined to extend to such a degree. We might suspect them to be comets about their aphelion, if the brightness as well as magnitude of the diameters did not oppose this idea; so that after all, we can hardly find any hypothesis so probable as that of their being Nebulæ; but then they must consist of stars that are compressed and accumulated in the highest degree. If it were not perhaps too hazardous to pursue a former surmise of a renewal in what I figuratively called the Laboratories of the universe, the stars forming these extraordinary nebulæ, by some decay or waste of nature, being no longer fit for their former purposes, and having their projectile forces, if any such they had, retarded in each other's atmosphere,

may rush at last together, and either in succession, or by one general tremendous shock, unite into a new body. Perhaps the extraordinary and sudden blaze of a new star in Cassiopeia's chair, in 1572, might possibly be of such a nature.

But left I should be led too far from the path of observation, to which I am resolved to limit myself, I shall only point out a considerable use that may be made of these curious bodies. If a little attention to them should prove that, having no annual parallax, they belong most probably to the class of nebulæ, they may then be expected to keep their situation better than any one of the stars belonging to our system, on account of their being probably at a very great distance. Now to have a fixed point somewhere in the heavens, to which the motions of the rest may be referred, is certainly of considerable consequence in Astronomy; and both these bodies are bright and small enough to answer that end[8].

[8]Having found two more of these curious objects, I add the place of them here, in hopes, that those who have fixed instruments may be induced to take an early opportunity of observing them carefully.
February 1, 1785. A very bright, planetary nebula, about half a minute in diameter, but the edges are not very well defined. It is perfectly round or perhaps a very little elliptical, and all over of a uniform brightness: with higher powers it becomes proportionally magnified. It follows γ Eridani $16'\,16''$ in time, and is $49'$ more north than that star.
February 7, 1785. A beautiful, very brilliant globe of light; a little hazy on the edges, but the haziness goes off very suddenly, so as not to exceed the 20th part of the diameter, which I suppose to be from 30 to $40''$. It is round, or perhaps a very little elliptical, and all over of a uniform brightness: I suppose the intensity of its light to be equal to that of a star of the ninth magnitude. It precedes the third b (Fl. 6,) Crateris $28'\,36''$ in time, and is $1^\circ\,25'$ more north than that star.

References

[1] J. Michell. XXVII. An inquiry into the probable parallax, and magnitude of the fixed stars, from the quantity of light which they afford us, and the particular circumstances of their situation. *Philosophical Transactions of the Royal Society of London*, 57:234–264, 1767. https://royalsocietypublishing.org/rstl/article/doi/10.1098/rstl.1767.0028/120122/XXVII-An-inquiry-into-the-probable-parallax-and.

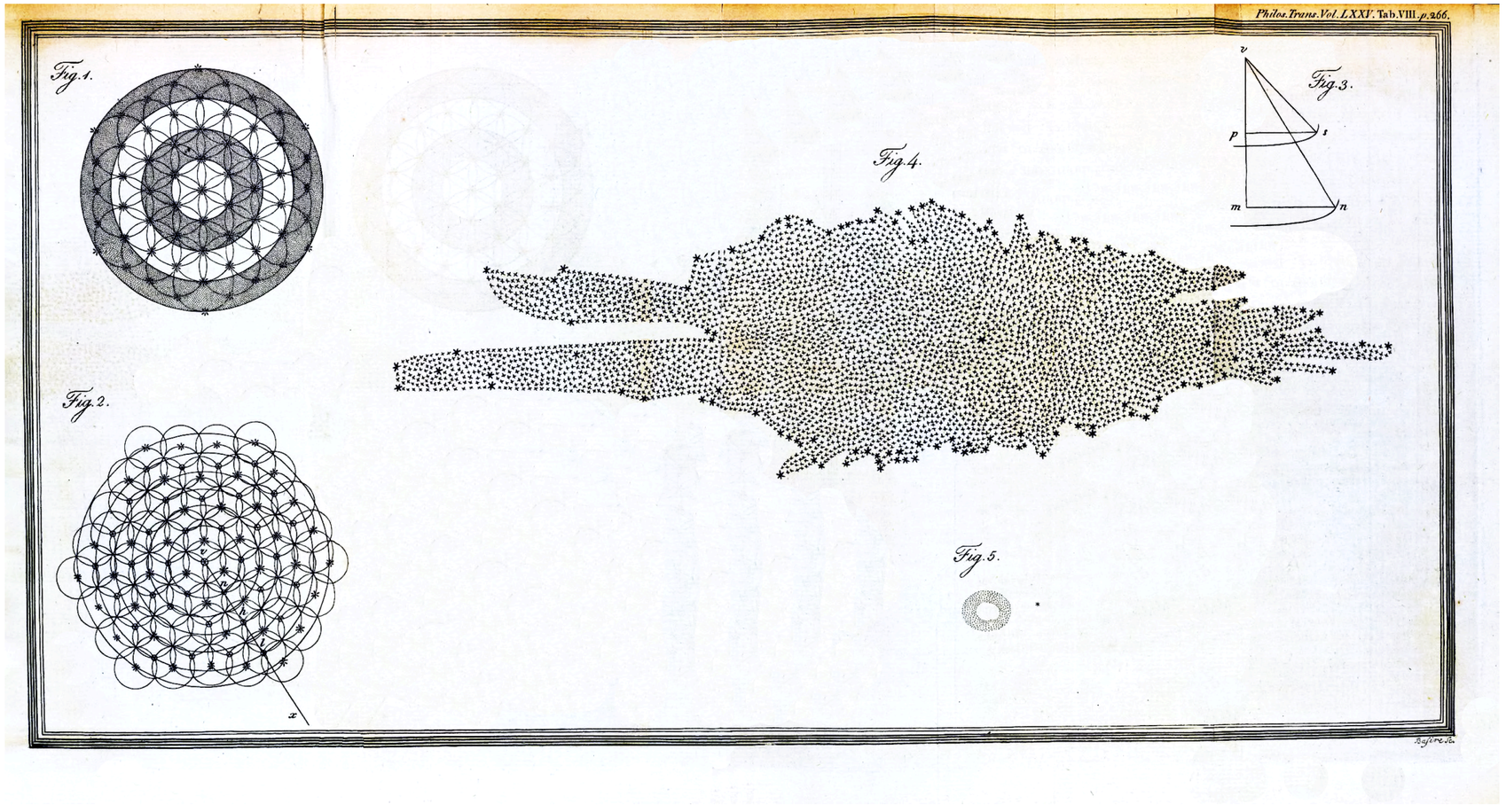

Figure 3: Tab. VIII, p. 266.